Mathematics: analysis and approaches
Higher Level
Paper 2

Name

Date: _____

2 hours

_____

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 15 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

The circle shown below has center O and radius measuring 4.25 cm.
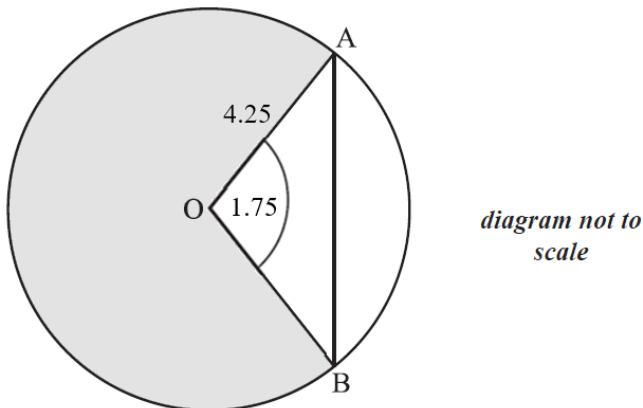
Points A and B lie on the circle and angle AOB measures 1.75 radians.

(a) Find AB. [3]

(b) Find the area of the shaded region. [4]

Question 1 continues on the next page

Question 1 continued

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2. [Maximum mark: 6]

A multiple-choice test consists of 12 questions. Each question has four answers from which to choose. Only one of the answers is correct. For each question, Boris randomly chooses one of the four answers.

(a) Write down the expected number of questions Boris answers correctly. [1]

(b) Find the probability that Boris answers exactly three questions correctly. [2]

(c) Find the probability that Boris answers more than three questions correctly. [3]

3. [Maximum mark: 6]

The diagram below shows part of the graph of the **gradient** function, $y = g'(x)$.
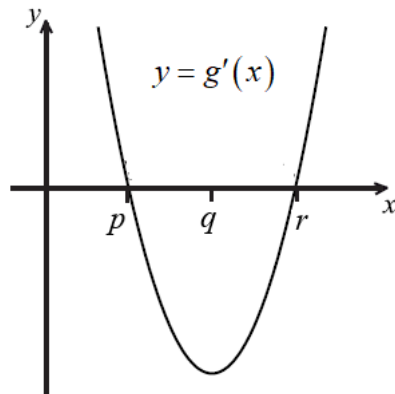
(a) On the grid below, sketch a graph of $y = g''(x)$, clearly indicating the x-intercept. [2]
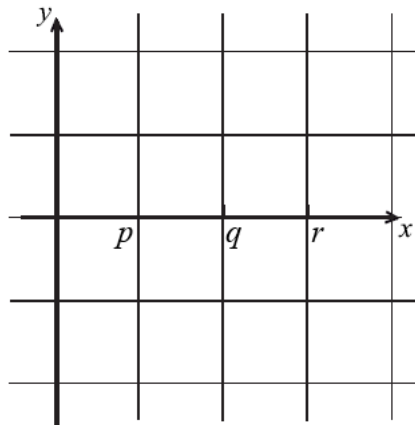
(b) Complete the table below, for the graph of $y = g(x)$. [2]

	x-coordinate
(i)　maximum point on g	
(ii)　minimum point on g	

(c) Justify your answer to part (b) (ii). [2]

4. [Maximum mark: 7]

Given that events A and B are independent, $\mathrm{P}(B) = 2\mathrm{P}(A)$, and $\mathrm{P}(A \cup B) = 0.72$, find $\mathrm{P}(B)$.
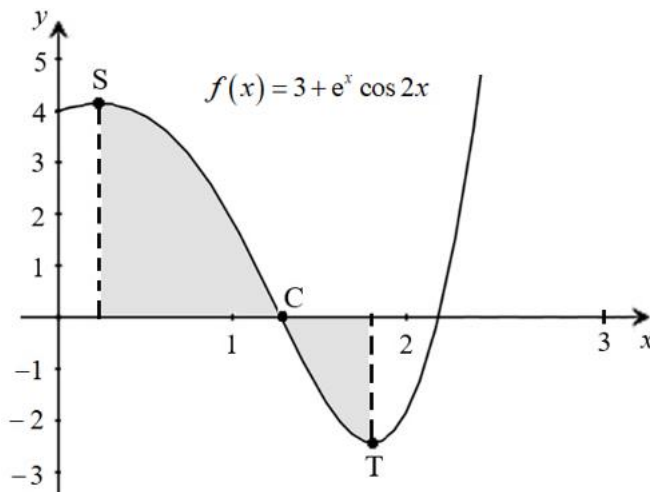
5. [Maximum mark: 6]

Let $f(x) = 3 + e^x \cos 2x$, for $0 \le x \le 3$. A portion of the graph of f is shown below.

There is an x-intercept at the point C, a local maximum point at S where $x = s$, and a minimum point at T where $x = t$.

(a) Write down the following:

    (i)   the x-coordinate of C;

    (ii)   the value of s;

    (iii)   the value of t.   [3]

(b) (i)   Let $\int_s^t f(x)\,dx = k$. Calculate the value of k.

    (ii)   Explain why k is **not** the area of the shaded region.   [3]

Question 5 continues on the next page

Question 5 continued

6. [Maximum mark: 5]

Given that $c > 0$, find the value(s) of x that solve the equation $|x+c| = |x| + c$.

7. [Maximum mark: 6]

The heights of sunflower plants in a large field can be modelled by a normal distribution. It is given that 60% of the plants are taller than 1.92 m and 25% are taller than 2.15 m. Find the mean and the standard deviation of the heights of the plants.

8.   [Maximum mark: 7]

Find the three cube roots of $4\sqrt{3} - 4i$ and express them exactly in exponential form, $r\,e^{i\theta}$.

9. [Maximum mark: 6]

The equations of three planes are given by
$$x + 2y + z = 3$$
$$-x + 2y + 3z = 1$$
$$-2x + y + 3z = a$$

(a) Find the value of a such that the three planes intersect in one line. [4]

(b) Find a vector equation for the line of intersection. [2]

Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 15]

A farmer has an operation growing button mushrooms indoors that are sold at a local market. On a particular day, the farmer harvests 200 button mushrooms and measures the diameter (d) of each mushroom in centimeters. The results are shown in the frequency table below.

diameter, d cm	$0<d\leq1$	$1<d\leq2$	$2<d\leq3$	$3<d\leq4$	$4<d\leq6$	$6<d\leq7.5$	$7.5<d\leq10$
frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the mean of the diameters of the mushrooms. [3]

(b) A cumulative frequency graph is given below for the diameters of the mushrooms.
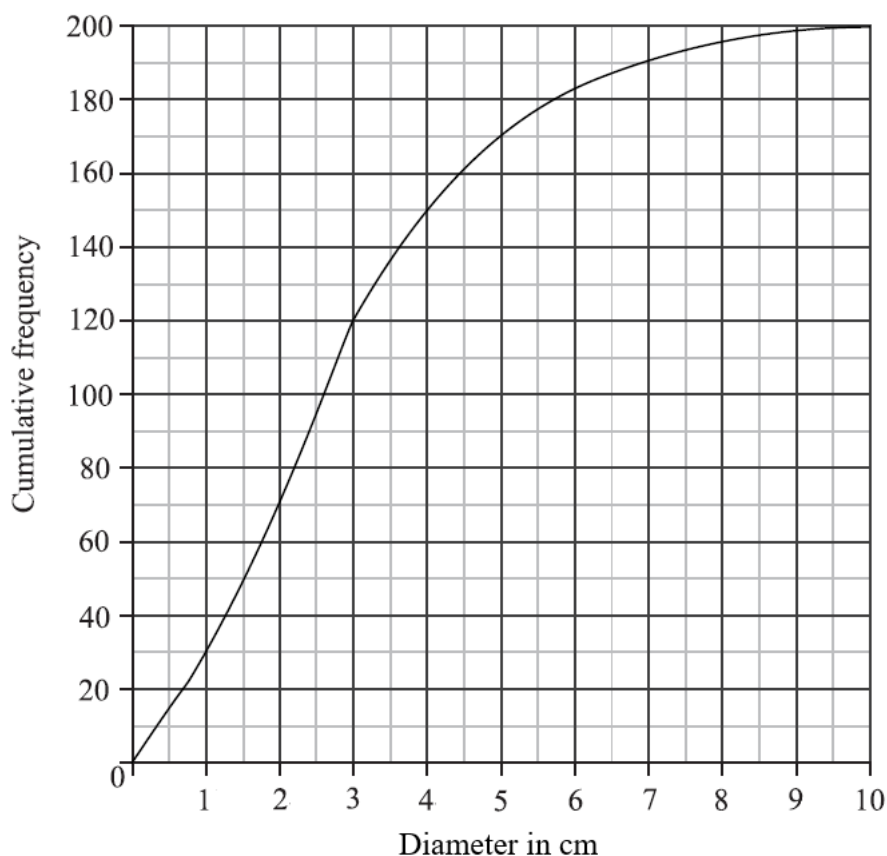
Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 20% of the mushrooms have a diameter more than k cm, find the value of k. [6]

Question 10 continues on the next page

Question 10 continued

In preparation for selling the mushrooms, the farmer classifies each of them as *small*, *medium* or *large* using the following criteria.

Small: diameter is less than 2 cm
Medium: diameter is greater than or equal to 2 cm but less than 6 cm
Large: diameter is greater than or equal to 6 cm

(c) Write down the probability that a mushroom randomly selected from the day's harvest is *Small*. [2]

The cost of a *Small* mushroom is $\$0.10$, a *Medium* mushroom is $\$0.15$ and a *Large* mushroom is $\$0.25$.

(d) Copy and complete the table below which is the probability distribution for the cost $\$X$. [2]

Cost $\$X$	0.10	0.15	0.25
$\mathrm{P}(X = x)$		0.565	

(e) Find $\mathrm{E}(X)$. [2]

11. [Maximum mark: 23]

The Cartesian equation of line L_1 is $x - 5 = \dfrac{y+3}{-3} = \dfrac{z-4}{2}$ and the Cartesian equation of

line L_2 is $\dfrac{x-2}{2} = y + 1 = \dfrac{z-3}{-1}$.

(a) Lines L_1 and L_2 intersect at point P. Find the coordinates of P. [5]

(b) Point Q is the point on L_1 that is nearest to the origin. Find the **exact** coordinates of Q. [6]

(c) Determine a Cartesian equation for the plane that contains L_1 and L_2. [4]

(d) Find the degree measure of the acute angle between the lines L_1 and L_2. [4]

(e) Line L_3 has the vector equation $\vec{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$. Find the values of k such that the

angle between L_2 and L_3 is $60°$. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 16]

(a) Show that $\dfrac{1}{4-x^2}$ can be expressed as $\dfrac{1}{4(x+2)} - \dfrac{1}{4(x-2)}$. [4]

(b) Hence, find $\displaystyle\int \dfrac{1}{4-x^2}\,dx$. [4]

The region R is bounded by the graph of $h(x) = \dfrac{1}{4-x^2}$ and the line $y = \dfrac{4}{7}$.

(c) Find the **exact** area of R. [5]

(d) The line $y = m$, where $m \in \mathbb{R}$, divides R into two regions of equal area. Write an equation whose solution is the value of m. Do **not** solve the equation. [3]